# TNM079: Modeling & Animation

Laboratory report

**Implement Surfaces and Modeling**

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# Contents

**Abstract**

This paper discuss on the method of represent implicit surfaces instead of explicit surfaces. In addition introduce method to implement Boolean operator between two surfaces and finally implementation of discrete gradient and curvature.

# 1. Introduction

Represent a surface is one of most important part of computer graphics world. The surfaces can divided into two main categories:

- Explicit
- Implicit

Explicit surface can represent like a mesh with triangles, edges and vertices but an implicit surface can represent by an equation and in order to find the geometry of the surface the equation should be solved.

## 1-1. Implicit Surfaces

Equation 1 shows the function which represent a circle.

$$f(x,y) = x^2 + y^2$$

Equation 1 – A function which represents a circle.

By solving equation 1 where $f(x,y) = C$ where $= R^2$ , the set of point of $(x,y)$ will be found which represents a sphere with radius of $R$.

Equation 2 shows the method to categorize all the point in a space.

$$\begin{gathered} Inside\ if\ f(x) < C \\ Outsid\ if\ f(x) > C \\ On\ surface\ if\ f(x) = C \end{gathered}$$

Equation 2 – Classification of points of a surface.

It will be used later to find the point which represent the surface.

### 1-2. Quadrics Surfaces

A quadric surface can be represented as equation 3.

$$f(x,y,z) = Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + 2Iz + J$$

Equation 3 – Function which represents a quadric surface.

Or it can be represented in a matrix form as equation 4.

$$P^tQP = [x\ y\ z\ 1]\begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Equation 4 – Function which represents a quadric surface as a matrix form.

## 2. Method

In this part method of implementation the Boolean operator, the quadric surface and discrete gradient and curvature operator for implicit surfaces will be described step by step:

## 2-1. Implement of CSG operator

The main three Boolean operators for an implicit surface should be implemented in this part.

These three Boolean operators are:

- Union
- Intersection
- Difference

These three Boolean operators can be shown as equation 5-7.

$$Union(A, B) = A \cup B = \min(A, B)$$

Equation 5 – The union of two primitives A and B.

$$Intersection(A, B) = A \cap B = \max(A, B)$$

Equation 6 – The intersection of two primitives A and B.

$$Difference(A, B) = A - B = \max(A, -B)$$

Equation 7 – The difference of two primitives A and B.

By using equation 5-7, just should call the *getvalue(x,y,z)* and use the *std::min* and *std::max* function in CGS.h file.

Do not forget to transfer all the points from world space to object space by using *Implicit::Transformw2o().*

## 2-2. Implement the quadric surface

Equation 4 shows the general function to calculate a quadric surface. A-I in the matrix are the coefficient which be constant for each quadric surface. Equation 8-13 shows the different coefficient for different quadric surface.

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \end{bmatrix}$$

Equation 8 – coefficient for a plane.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Equation 9 – coefficient for a cylinder.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Equation 10 – coefficient for a sphere.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation 11 – coefficient for a cone.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & -0.5 & 0 \end{bmatrix}$$

Equation 12 – coefficient for a paraboloid.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation 13 – coefficient for a hyperboloid.

By using equation 14, the calculation of gradient operator for a quadric surface will be done.

$$\nabla f(x,y,z) = 2 \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Equation 14 – Calculation for Quadric gradient.

## 2-3. Implement the discrete gradient operator for implicit

Equation 15 shows the calculation to implement the discrete gradient operator.

$$D_x = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \approx \frac{f(x_0 + \mathcal{E}) - f(x_0 - \mathcal{E})}{2\mathcal{E}}$$

Equation 15 – Calculation of discrete gradient.

This equation should be applied three times to get the final result. So it can be written as equation 16.

$$D_x = \frac{f(x + \mathcal{E}, y, z) - f(x - \mathcal{E}, y, z)}{2\mathcal{E}}$$

$$D_y = \frac{f(x\,, y + \mathcal{E}\,, z) - f(x\,,\; y - \mathcal{E}\;, z)}{2\mathcal{E}}$$

$$D_z = \frac{f(x\,, y\,, z + \mathcal{E}) - f(x\,,\; y\,, z - \mathcal{E}\,)}{2\mathcal{E}}$$

Equation 16 – Calculation of discrete gradient for x, y and z.

It is clear to calculate $f(x + \mathcal{E}\,, y\,, z)$, *getvalue(*$x + \mathcal{E}\,, y\,, z$*)* can be used.

## 2-4. Implement the discrete curvature operator for implicit

Equation 17 shows the calculation to implement the discrete gradient operator.

$$\mathrm{K} = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2} + \frac{d^2 f}{dz^2}$$

Equation 17 – Calculation of discrete curvature

This equation can be written as equation 18.

$$D_{xx} = \frac{f(x + \mathcal{E}\,) - 2f(x\;\,) + f(x - \mathcal{E}\,)}{\mathcal{E}^2}$$

Equation 18 – Calculation $D_{xx}$ in order to calculate the discrete curvature.

Do not forget in this section and the previous one, $\mathcal{E}$ is represent by the *mDelta* variable that gets its value directly from the interface in runtime.

## 2-5. Implement super-elliptic blending

By super-elliptic blending, smoother Boolean operation can be expected. Union, intersection and difference can be shown as equation 19-21.

$$D_{A\cup B} = ({D_A}^p + {D_B}^p)^{\frac{1}{p}}$$

Equation 19 – The union of two primitives A and B – Super-elliptic blending.

$$D_{A\cap B} = ({D_A}^{-p} + {D_B}^{-p})^{-\frac{1}{p}}$$

Equation 20 – The intersection of two primitives A and B – Super-elliptic blending.

$$D_{A-B} = ({D_A}^p - {D_B}^p)^{-\frac{1}{p}}$$

Equation 21 – The difference of two primitives A and B – Super-elliptic blending.

Before using these equations, the surface should be converted into a density form. It is simply done by using equation 22.

$$D_{A(x)} = e^{-A(x)}$$

Equation 22 – convert implicit surface to density.

It is important that after the calculation, by using equation 23, back the density to its surface.

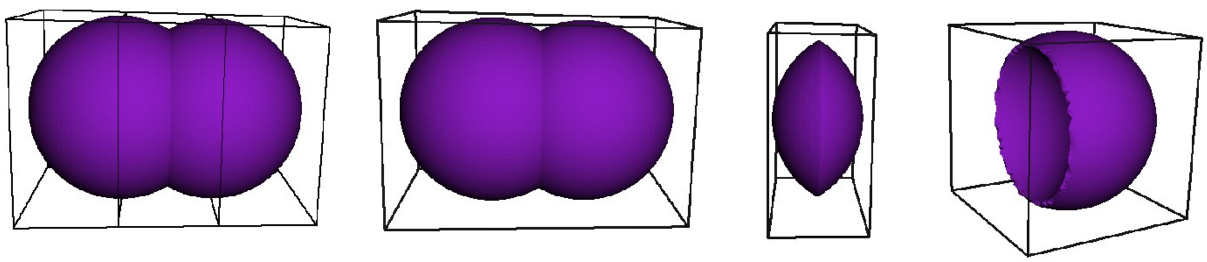
$$\mathrm{F(x)} = -\log(D_{A(x)})$$

Equation 23 – convert density to implicit surface.

# 3. Result

## 3-1. Implement of CSG operator

By implement the CGS operator, union, intersection and difference between two implicit faces can be found easily. Fig. 1 shows these three CGS operators between two implicit spheres.

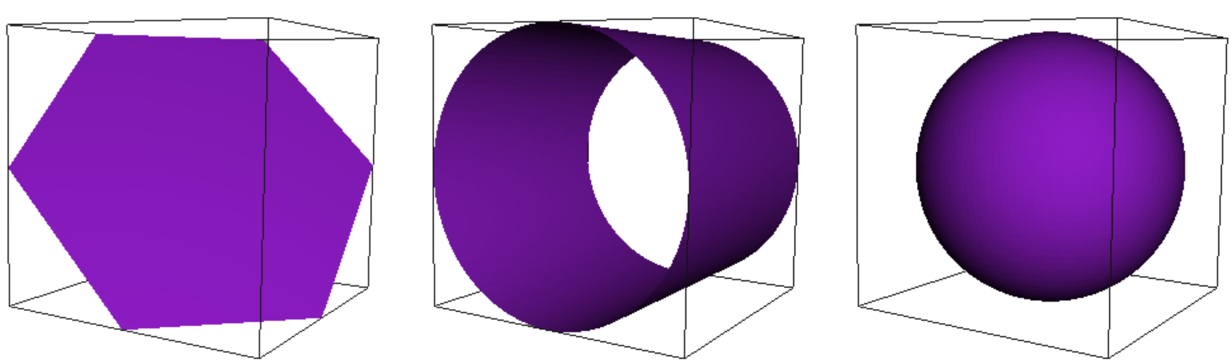


Two sphere before any operation — union — intersection — difference

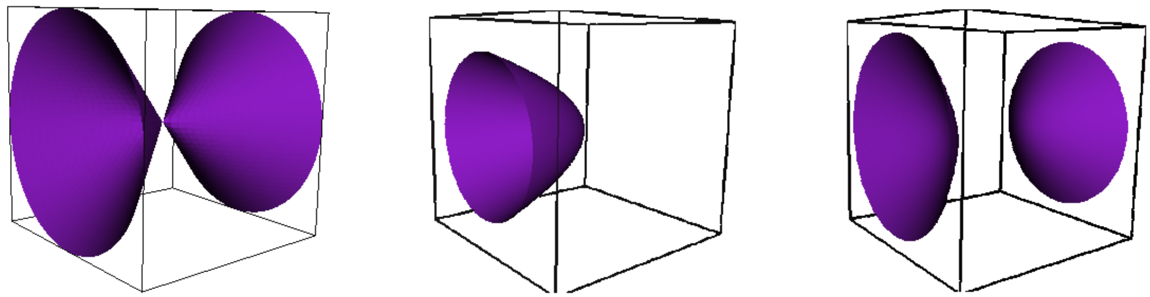
**Fig 1:** Implement of CSG operator

## 3-2. Implement the quadric surface

Different quadric surfaces are shown in Fig. 2.



Plane — Cylinder — Sphere



Cone paraboloid Hyperboloid

**Fig 2:** Different quadric surfaces.

Fig. 3 shows the gradient of a quadric surface.

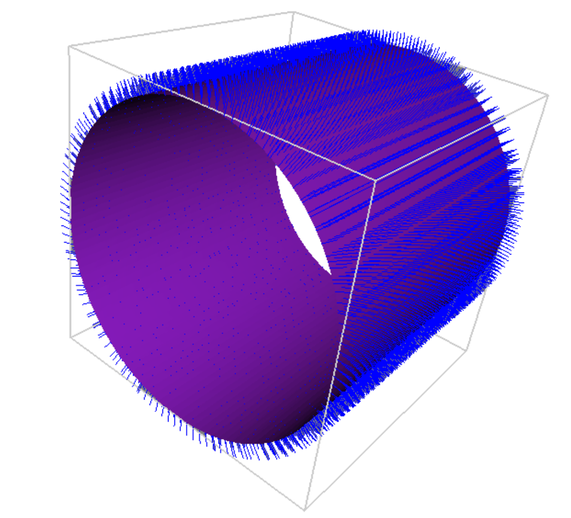


**Fig 3:** Gradient of a quadric surface.

## 3-3. Implement the discrete gradient operator for implicit

As explained before, discrete gradient can be calculated by Equation 15. Fig. 4 shows the gradient of an implicit sphere.

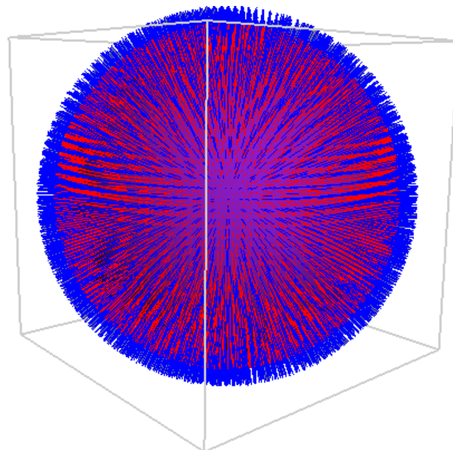


**Fig 4:** discrete gradient for implicit sphere. The blue lines are gradient and the red lines are face normal.

As it clear in the Fig. 3 the discrete gradient are parallel with the face normal.

### 3-4. Implement super-elliptic blending

Fig. 5 shows super-elliptic blending for a sphere with different blended value.

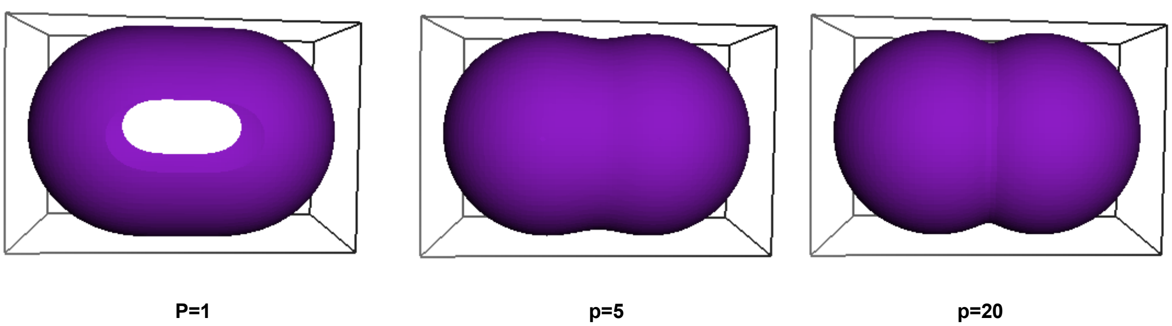



**Fig 5:** Union Super-elliptic blending.

## 4. Conclusion

Using implicit surfaces helps to represent complex surfaces with a good resolution. Also by using Boolean operators, it is easy to generate new and more complex surfaces.

Since I have completed all the * and **, I should get grade5.